CENTER FOR AERONAUTICAL RESEARCH
THE UNIVERSITY OF TEXAS AT AUSTIN

Bureau of Engineering Research
The University of Texas at Austin
Austin, Texas

CAR 85-3

Multishaker Modal Testing

Roy R. Craig, Jr.

Final Report
NASA Contract No. NAS8-35338

May 1985

MULTISHAKER MODAL TESTING

Final Report
Contract No. NAS8-35338

August 1983 - May 1985

Prepared for

National Aeronautics and Space Administration
George C. Marshall Space Flight Center
Marshall Space Flight Center, AL 35812

by

Dr. Roy R. Craig, Jr.
Professor, ASE-EM
Principal Investigator

Center for Aeronautical Research
Bureau of Engineering Research
The University of Texas at Austin

May 1985

TABLE OF CONTENTS

ABSTRACT

This report summarizes the research conducted on this contract and previously reported on in a number of technical reports, papers presented at technical meetings, and published papers. The goals of the contract were to develop a component mode synthesis method for damped structures and to explore modal test methods which could be employed to determine the relevant parameters required by the component mode synthesis method.

Research was conducted on the following topics:

1) Development of a generalized time-domain component mode synthesis technique for damped systems,
2) Development of a frequency-domain component mode synthesis method for damped systems, and
3) Development of a system identification algorithm applicable to general damped systems.

This report presents abstracts of the major publications which have been previously issued on these topics.

TIME-DOMAIN COMPONENT MODE SYNTHESIS FOR DAMPED SYSTEMS

Under a previous contract, NAS8-33980, between The University of Texas at Austin and NASA Marshall Space Flight Center, work was begun on the development of component mode synthesis techniques for damped structures. The Final Report for that contract (1) summarizes work by Chung and Craig and by Howsman and Craig. The work of Howsman, which is based on a state-vector component mode synthesis formulation, was continued under the present contract, and a report (Ref. 2) covering that research is included as an adjunct to this Final Report.

FREQUENCY-DOMAIN COMPONENT MODE SYNTHESIS FOR DAMPED SYSTEMS

Time-domain component mode synthesis techniques transform the component differential equations of motion using component modes. Interface compatibility is invoked to couple the transformed component equations to form a coupled set of system differential equations. These can subsequently be solved for system modes and frequencies or integrated directly. On the other hand, frequency-domain analysis involves a coupling of dynamic stiffness matrices obtained by transforming the component differential equations of motion to the frequency domain by use of the Fourier transform.

Although frequency-domain analysis is not nearly as common as time-domain analysis, frequency-domain techniques have been used for many years for determining the steady-state response of structures subjected to periodic excitation. However, very little work has been done on the topic of determining the transient response of structures using frequency domain techniques. Since modal testing of structures frequently results in a set of frequency response functions describing the output/input behavior of the structures, a study has been conducted to determine efficient frequency-domain techniques for computing transient response of coupled structures. This work is reported on in Ref. 3, which is included as an adjunct to this Final Report.

SYSTEM IDENTIFICATION OF DAMPED STRUCTURES

Under a previous contract, NAS8-33980, between The University of Texas at Austin and NASA Marshall Space Flight Center, Craig and Blair developed a generalized multiple-input, multiple-output modal parameter estimation algorithm (Ref. 4). This frequency-domain modal parameter estimation algorithm has been extended by Kurdila and Craig to be applicable to systems with general linear viscous damping. Automated procedures for identifying "independent" and "dependent" measurement stations are included in the algorithm. The algorithm is presented in Ref. 5, which is incorporated as an adjunct to this Final Report.

ABSTRACTS OF TECHNICAL REPORTS

A Substructure Coupling Procedure Applicable to General Linear Time-Invariant Dynamic Systems (Ref. 2)

The substructure coupling procedure presented in this report is valid for systems possessing general nonproportional, even nonsymmetric, damping. The coupled system equations of motion are derived from a variational principle, and free-interface component modes, along with a set of attachment modes serve as the substructure Ritz vectors, or assumed modes. A brief summary of the theory presented in this report follows.

The substructure equation of motion is written in the standard matrix second-order form

$$m\ddot{x} + c\dot{x} + kx = f \tag{1}$$

where it is assumed that the coefficient matrices may be nonsymmetric. It can be demonstrated that the adjoint equation corresponding to Eq. (1) is

$$m^T\ddot{y} - c^T\dot{y} + k^Ty = f^* \tag{2}$$

To facilitate the solution of Eq. (1), it may be cast into state variable form by defining a velocity variable v through the equation

$$m(\dot{x} - v) = 0 \tag{3}$$

Using variational methods, it is shown that the standard and adjoint differential equations may be written in state variable matrix form as

$$A\dot{X} + BX = F \tag{4a}$$

$$-A^T\dot{Y} + B^TY = F^* \tag{4b}$$

where

$$A = \begin{bmatrix} 0 & m \\ m & c \end{bmatrix} \quad , \quad B = \begin{bmatrix} -m & 0 \\ 0 & k \end{bmatrix}$$

$$X = \begin{Bmatrix} v \\ x \end{Bmatrix}, \quad Y = \begin{Bmatrix} w \\ y \end{Bmatrix}, \quad F = \begin{Bmatrix} 0 \\ f \end{Bmatrix}, \quad F^* = \begin{Bmatrix} 0 \\ f^* \end{Bmatrix}$$

One of the key motivations for substructure coupling is the possibility of systematicaly reducing the system order by introducing Ritz approximations to the substructure state variable vectors in the form

$$X = \Phi_x \; \eta_x \tag{5a}$$

$$Y = \Phi_y \; \eta_y \tag{5b}$$

where Φ_x and Φ_y may contain, respectively, right and left substructure eigenvectors or other Ritz vectors.

Coupling of substructures can be illustrated by considering two substructures α and β whose interface compatibility can be expressed by

$$E^{\alpha} X^{\alpha} = E^{\beta} \; X^{\beta} \tag{6}$$

A similar equation holds for Y. Equations (5) and (6) may be combined, and the generalized coordinates separated into independent and dependent coordinates to produce coupling transformation matrices C_x and C_y such that

$$\begin{Bmatrix} \eta_{xI} \\ \eta_{xD} \end{Bmatrix} = C_x \eta_{xI} \quad , \quad \begin{Bmatrix} \eta_{yI} \\ \eta_{yD} \end{Bmatrix} = C_y \eta_{yI} \tag{7}$$

The independent system equations of motion can finally be written in the form

$$A_s \dot{\eta}_s + B_s \eta_s = F_s \tag{8}$$

where

$$A_s = C_y^T \Phi_{ys}^T A_{bd} \Phi_{xs} C_x$$

$$B_s = C_y^T \Phi_{ys}^T B_{bd} \Phi_{xs} C_x$$

$$F_s = C_y^T \Phi_{ys}^T F$$

$$\eta_s = \eta_{xI}$$

and where A_{bd} and B_{bd} are block-diagonal matrices

$$A_{bd} = \begin{bmatrix} A^\alpha & 0 \\ 0 & A^\beta \end{bmatrix} , \quad B_{bd} = \begin{bmatrix} B^\alpha & 0 \\ 0 & B^\beta \end{bmatrix} , \quad F = \begin{Bmatrix} F^\alpha \\ F^\beta \end{Bmatrix}$$

The adjoint system equations of motion can be formed in a similar manner.

The above substructure coupling procedure is very straightforward and is applicable to systems with any form of linear damping. The "price" is the use of state vectors and the requirement for adjoint Ritz vectors if biorthogonality is used to diagonalize the substructure state matrices. However, the adjoint problem need not be solved otherwise. Free-interface component modes and residual attachment modes form the Ritz transformations Φ_x and Φ_y.

Substructure Coupling in the Frequency Domain (Ref. 3)

This report explores procedures for coupling frequency-domain component models and using the synthesized frequency-domain system model to compute transient response. The contents of this report are briefly summarized below.

Let the dynamic substructure stiffness matrices be defined by

$$G^i(s) = m^i s^2 + c^i s + k^i \quad , \quad i = \alpha,\beta \tag{9}$$

Therefore, the Laplace-transformed equations of motion for the ith substructure may be written

$$\begin{bmatrix} G^i_{JJ} & G^i_{JE} \\ G^i_{EJ} & G^i_{EE} \end{bmatrix} \begin{Bmatrix} X^i_J \\ X^i_E \end{Bmatrix} = \begin{Bmatrix} F^i_J + F^i_C \\ F^i_E \end{Bmatrix} \tag{10}$$

where J denotes juncture (interface) coordinate, E denotes external coordinate, $F_C^{\ i}$ denotes interface forces due to adjacent substructures, and $F_j^{\ i}$ and $F_E^{\ i}$ are external forces. Displacement compatibility and force equilibrium at the interface are given by

$$X^\alpha_J = X^\beta_J = X_j \tag{11a}$$

and

$$F^\alpha_C + F^\beta_C = 0 \tag{11b}$$

respectively. Equations (10) and (11) can be combined to form the system equation of motion

$$\begin{bmatrix} G^\alpha_{EE} & 0 & G^\alpha_{EJ} \\ 0 & G^\beta_{EE} & G^\beta_{EJ} \\ G^\alpha_{JE} & G^\beta_{JE} & (G^\alpha_{JJ} + G^\beta_{JJ}) \end{bmatrix} \begin{Bmatrix} X^\alpha_E \\ X^\beta_E \\ X_J \end{Bmatrix} = \begin{Bmatrix} F^\alpha_E \\ F^\beta_E \\ F^\alpha_J + F^\beta_J \end{Bmatrix} \tag{12}$$

Equation (12) can be written in more compact form as

$$G(s)\,X(s) = F(s) \tag{13}$$

Its solution in the Laplace domain is thus

$$X(s) = H(s)\,F(s) \tag{14}$$

where

$$H(s) = [G(s)]^{-1} \tag{15}$$

Equation (14) may be employed for computing steady-state response by letting $s = j\omega$. However, further consideration is required if a transient (time domain) solution is required. In this report, Laplace-domain equations are considered as "offset" frequency-domain equations, and a discrete Fourier transform (DFT) technique is employed to compute transient solutions.

In Eq. (13) let

$$X(s) = \int_0^\infty \hat{x}(t)\, e^{-j\omega t}\, dt \tag{16}$$

and

$$F(s) = \tilde{F}(s) + M\dot{x}_0(s) + [Ms + C]\, x_0(s) \tag{17}$$

where

$$s = a + j\omega \tag{18}$$

and

$$\tilde{F}(s) = \int_0^\infty f(t)\, e^{-at}\, e^{-j\omega t}\, dt \tag{19}$$

Then, the transient response $x(t)$ is obtained from Eqs. (14) and (16) by inverse Fourier transforming Eq. (16) to obtain $\hat{x}(t)$ and then noting that

$$x(t) = \hat{x}(t)\, e^{at} \tag{20}$$

Reference 3 contains examples which illustrate the selection of appropriate computational parameters, such as the Nyquist frequency for frequency discretization and the convergence factor "a" in e^{-at}.

A Modal Parameter Extraction Procedure Applicable to Linear Time-Invariant Dynamic Systems (Ref. 5)

In this report, a new modal parameter estimation procedure is presented. The technique is applicable to linear, time-invariant systems. It represents a further development of the multiple-input, multiple-output algorithm presented by Craig and Blair (4), which is an extension of Coppolino's Simultaneous Frequency Domain (SFD) technique (6). The principal contributions of the research described in this report are:

(1) the incorporation of automatic procedures to reduce effective problem size,

(2) the incorporation of general (linear) damping, rather than proportional damping, and

(3) the use of more stable solution procedures than those employed by Coppolino (6) and by Craig and Blair (4).

In developing a modal parameter estimation method for linear, time-invariant systems, it is assumed that there exists an analytical model which represents the dynamics of the structure and which is given by

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = f(t) \tag{21}$$

where $\ddot{x}(t)$, $\dot{x}(t)$, and $x(t)$ are the acceleration, velocity and displacement histories at N discrete locations on the structure. This model is reduced to a smaller order, which depends on the number of active modes in the frequency band of interest. The process of reducing, and later reconstructing, a set of time histories is embodied in the following equations:

$$\gamma(t) = \Psi_C \, x(t) \tag{22}$$

$$x(t) \cong \Psi_R \, \gamma(t) \tag{23a}$$

$$x(t) = \Psi_R \, \gamma(t) + \varepsilon(t) \tag{23b}$$

where $\gamma(t)$ is the vector of "condensed" coordinates, and where Ψ_C is the condensing transformation and Ψ_R the reconstructing transformation. One chapter of Ref. 5 is devoted to automatic procedures for determining Ψ_C and Ψ_R.

Equations (21) through (23) may be combined and the resulting equation Fourier-transformed to give the reduced frequency domain equation

$$M^{*}\ddot{\gamma}(w) + C^{*}\dot{\gamma}(w) + K^{*}\gamma(w) = \Psi_C D \, p(w) \tag{24}$$

where D is a force distribution matrix and where

$$\begin{aligned} M^{*} &= \Psi_C M \Psi_R \\ C^{*} &= \Psi_C C \Psi_R \\ K^{*} &= \Psi_C K \Psi_R \end{aligned} \tag{25}$$

Frequency-domain relationships among $\ddot{\gamma}$, $\dot{\gamma}$, and γ may be combined with Eq. (24) to give the final reduced frequency-domain equation

$$\ddot{\gamma}(w) + (1/jw)\hat{C}\ddot{\gamma} - (1/w^2)\hat{K}\ddot{\gamma} = \hat{D} \, p(w) \tag{26}$$

Least-squares solutions based on singular-value decomposition and on Householder transformations are employed to determine the reduced matrices $\hat{C}$, $\hat{K}$, and $\hat{D}$. Finally, $\hat{C}$ and $\hat{K}$ are used as coefficient matrices in an eigenproblem whose solution gives the natural frequencies, damping factors, and mode shapes of the system.

Example problems are presented to illustrate the stability of the algorithm, its ability to handle non-proportional damping and closely-spaced-frequency modes, and the techniques employed for selection of the condensing transformation and for solution of the least-squares problem.

ABSTRACTS OF TECHNICAL PAPERS

A Generalized Multiple-Input, Multiple-Output Modal Parameter Estimation Algorithm (Ref. 4)

This paper, which has been accepted for publication in the AIAA Journal, describes a multiple-input, multiple-output frequency-domain modal parameter estimation algorithm. The algorithm is restricted to systems with proportional damping, but is otherwise very similar to the more general procedure described in Ref. 5, which is abstracted above.

Some Approaches to Substructure Coupling with Damping (Ref. 7)

This paper, which was presented orally at the 4th International Conference on Applied Numerical Modeling in Tainan, Taiwan, has also been submitted for publication in the Journal of Astronautical Sciences. It is condensed from Refs. 2 and 3, which are abstracted above, and shows the relationships between time-domain and frequency-domain approaches to substructure coupling. That relationship is summarized in Figure 1.

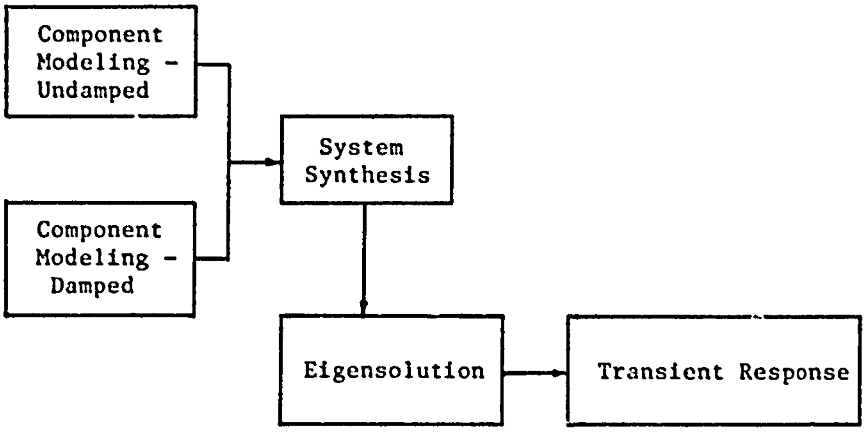

Fig. 1a. Time-Domain Transient Response Solution

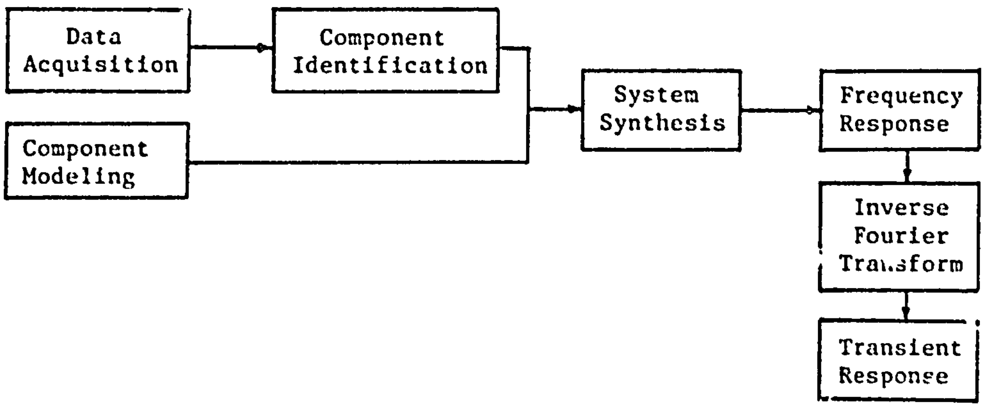

Fig. 1b. Frequency-Domain Transient Response Solution

Modal Vector Estimation for Closely-Spaced-Frequency Modes (Ref. 8)

This paper is based on a report (9) published under the preceeding contract, NAS8-33980. However, as part of the continuing study of Multishaker Modal Testing, its presentation at an international technical meeting and inclusion in the proceedings of that meeting are within the scope of the present contract.

Complete derivations of the Asher method and the minimum coincident response method of modal tuning are presented in the paper. Both methods were applied to determine the mode shapes of a weakly-coupled structure and to illustrate the use of curve-fitting of experimental frequency response functions to obtain input data for the tuning process. The paper concludes that (1) the Asher method and the minimum coincident response method are rational procedures for employing multiple FRF's for identifying modal parameters, and (2) mode shapes of lightly-coupled systems are extremely sensitive to changes in system parameters and are therefore difficult to identify uniquely.

EQUIPMENT ACQUISITION

The following equipment was purchased for use in the research on multishaker modal testing covered by this contract.

8	channels of data acquisition hardware and large-screen monitor to augment Genrad 2515 computer Aided Test System	$14320.15
4	5-lb electrodynamic shakers	5736.00
1	impact hammer including force cell	628.47
	TOTAL	$ 20,684.62

CONCLUSIONS AND RECOMMENDATIONS

Several significant contributions are contained in the reports and papers prepared under this contract and abstracted above. The principal contributions are:

1. The development of a component mode synthesis method for systems with general viscous damping has been completed. By incorporating residual attachment modes as well as free-vibration modes, the convergence rate is improved.

2. Methods of component mode synthesis in the frequency domain have been explored, and it has been demonstrated that digital Fourier transforms (DFT's) may be employed to compute transient response of a coupled structure.

3. A multi-input, multi-output modal parameter estimation algorithm has been developed which is capable of identifying frequencies, damping values, and mode shapes of systems with general viscous damping. The algorithm is robust and requires little user interaction.

The following recommendations are made for further work:

1. Component mode synthesis procedures for damped structures which are more efficient than the ones currently available, including Ref. 2, should be developed.

2. The application of frequency-domain techniques to compute transient response of coupled structures should be explored further. In particular, the techniques developed in Ref. 3 should be applied to systems having more degress of freedom. The efficiency of such a frequency-domain approach should be compared with time-domain methods for computing transient response of coupled structures.

3. The modal parameter estimation algorithm developed in Ref. 5 is applicable to general linear, time-invariant dynamic systems. It results in reduced system matrices, which might be very useful in an application such as control of flexible structures. Applications of the algorithm should be

explored, and it should be compared with other modal parameter estimation algorithms, such as the Polyreference algorithm.

SELECTED REFERENCES

1. Craig, R. R., Jr., Multishaker Modal Testing, Report No. CAR 83-2, Center for Aeronautical Research, Bureau of Engineering Research, The University of Texas at Austin, October 1983.

2. Howsman, T. G., and Craig, R. R., Jr., A Substructure Coupling Procedure Applicable to General Linear Time-Invariant Dynamic Systems, Report No. CAR 84-1, Center for Aeronautical Research, Bureau of Engineering Research, The University of Texas at Austin, May 1984.

3. Bachmeyer, R. C., and Craig, R. R., Jr., Substructure Coupling in the Frequency Domain, Report No. CAR 85-1, Center for Aeronautical Research, Bureau of Engineering Research, The University of Texas at Austin, May 1985.

4. Craig, R. R. Jr., and Blair, M. A., "A Generalized Multiple-Intput, Multiple-Output Modal Parameter Estimation Algorithm," accepted for publication in AIAA Journal.

5. Kurdila, A. J., and Craig, R. R., Jr., A Modal Parameter Extraction Procedure Applicable to Linear Time-Invariant Dynamic Systems, Report No. CAR 85-2, Center for Aeronautical Research, Bureau of Engineering Research, The University of Texas at Austin, May 1985.

6. Coppolino, R. N., "A Simultaneous Frequency Domain Technique for Estimation of Modal Parameters from Measured Data," SAE Paper No. 811046, Oct. 1981.

7. Craig, R. R., Jr., Bachmeyer, R. C., and Howsman, T. G., "Some Approaches to Substructure Coupling with Damping," Proc. 4th International Conf. on Applied Numerical Modeling, Tainan, Taiwan, Dec. 1984, pp. 172-177.

End of Document